Annals of Process Engineering and Management

www.apem.reapress.com

Ann. Proc. Eng. Manag. Vol. 3, No. 2 (2026) 67-89.

Paper Type: Original Article

Upside-Down Logic within Plithogenic Fuzzy ITIL: A Framework for Context-Aware Contradiction Handling in IT Service Management

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Citation:

Received: 20 September 2025 *Revised: 23 December 2025* *Accepted: 15 February 2026*	Fujita, T. (2026). Upside-down logic within plithogenic fuzzy ITIL: A framework for context-aware contradiction handling in IT service management. *Annals of process engineering and management, 3*(2), 67-89.

Abstract

In the real world, many reversal phenomena occur—for instance, situations where a statement once considered false is later recognized as true. *Upside-Down Logic* provides a framework to formalize such reversal phenomena as a logical system. It reverses the truth and falsity of propositions through contextual transformations, thereby capturing ambiguity and reversals within reasoning processes. A *Plithogenic Set* represents elements using attribute-based membership and contradiction functions, extending the traditional frameworks of fuzzy, intuitionistic, and neutrosophic sets. Information Technology Infrastructure Library (ITIL) is a framework of best practices for IT service management, focusing on the service lifecycle, efficiency, alignment, and continual improvement. Extensions of ITIL have been studied through fuzzy set theory, such as the *Fuzzy ITIL*. In this paper, we investigate *Plithogenic Fuzzy ITIL*, which extends Fuzzy ITIL by incorporating the structure of a Plithogenic Fuzzy Set.

Keywords: Upside-down-logic, Plithogenic set, De-plithogenication, ITIL, Plithogenic fuzzy ITI.

1| Introduction

This section gathers the basic notions and symbols used throughout the paper. Unless explicitly stated otherwise, all sets and structures are finite.

1.1| Formal Definition of Upside-Down Logic

We now give a precise formulation of Upside-Down Logic. Informally, it reverses the truth status of propositions under suitable context transformations, thereby capturing ambiguity and reversal phenomena in reasoning systems [1-6]. The required notation and concepts are introduced below.

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https://doi.org/10.48314/apem.vi.51

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Definition 1 (Context) ([2], [3]). A *context* $\mathcal{C}$ is a specification of the parameters or conditions under which propositions are assessed. Typical components include spatial, temporal, semantic, or interpretive settings.

Definition 2 (Logical System) ([7]). A *logical system* is a structure

$$\mathcal{M} \;=\; (\mathcal{P},\, \mathcal{V},\, v),$$

where:

- $\mathcal{P}$ is the collection of propositions (well-formed statements) of a formal language $\mathcal{L}$;
- $\mathcal{V}$ is the set of truth values (e.g., $\{\text{True}, \text{False}\}$ in the classical case);
- $v : \mathcal{P} \to \mathcal{V}$ is a *valuation* assigning a truth value to each proposition.

Optionally, $\mathcal{M}$ may include:

- a set of *axioms* $\mathcal{A} \subseteq \mathcal{P}$ declared true within the system;
- a set of *inference rules* $\mathcal{I}$ specifying admissible derivations of new truths.

Notation 1. *Given a set of propositions $\mathcal{P}$ and a set of contexts $\mathcal{C}$, define the context-sensitive valuation*

$$T :\; \mathcal{P} \times \mathcal{C} \longrightarrow \{\textit{True},\, \textit{False},\, \textit{Indeterminate}\},$$

which returns the truth value of a proposition under a specified context.

Notation 2. *Let $\mathcal{L}$ be a formal language, and let $\mathcal{M} = (\mathcal{P}, \mathcal{V}, v)$ be a logical system with proposition set $\mathcal{P}$, truth-value set $\mathcal{V}$, and valuation $v : \mathcal{P} \to \mathcal{V}$.*

Definition 3 (Upside-Down Logic) ([2], [3]). An *Upside-Down Logic* is any system $\mathcal{M}'$ obtained from $\mathcal{M}$ by a transformation U acting on propositions and/or contexts such that:

(1) For every $A \in \mathcal{P}$ with truth value $v(A)$ in context $\mathcal{C}$, there exist a transformed proposition $U(A)$ and/or a transformed context $U(\mathcal{C})$ satisfying:

- *Falsification of the True:* if $v(A) = \text{True}$ in $\mathcal{C}$, then $v\big(U(A)\big) = \text{False}$ in $U(\mathcal{C})$.
- *Truthification of the False:* if $v(A) = \text{False}$ in $\mathcal{C}$, then $v\big(U(A)\big) = \text{True}$ in $U(\mathcal{C})$.

(2) The operator U is well defined and yields a coherent (internally consistent) system $\mathcal{M}'$.

Example 1 (Release gate flips under hotfix urgency (Falsification of the True))**.** *Proposition.* $A :=$ "Block the production release now." *Observable.* quantified risk $r = 0.08$ (dimensionless). *Policy thresholds.* $\theta_{\text{normal}} = 0.02$ (normal), $\theta_{\text{hotfix}} = 0.20$ (urgent hotfix). *Valuation rule.*

$$T(A, C) \;=\; \mathbf{1}\,[\, r \geq \theta_C \,].$$

In the *normal* context C_{normal}, $T(A, C_{\text{normal}}) \;=\; \mathbf{1}[0.08 \geq 0.02] \;=\; 1$ (True). Apply the context transform $U(C_{\text{normal}}) = C_{\text{hotfix}}$. Then

$$T\big(A,\, U(C_{\text{normal}})\big) \;=\; \mathbf{1}[0.08 \geq 0.20] \;=\; 0 \quad (\text{False}).$$

Thus a previously true gate (*block*) becomes false under the hotfix policy—an instance of *Falsification of the True*.

Example 2 (Paging suppression during a maintenance window (Falsification of the True))**.** *Proposition.* $A :=$ "Page the on-call immediately." *Observable.* synthetic-check failure ratio $s = 0.90$. *Paging thresholds.* $\tau_{\text{normal}} = 0.20$ (normal), $\tau_{\text{maint}} = 0.95$ (scheduled maintenance). *Valuation rule.*

$$T(A, C) \;=\; \mathbf{1}\,[\,s \geq \tau_C\,].$$

Normal context: $T(A, C_{\text{normal}}) = \mathbf{1}[0.90 \geq 0.20] = 1$ (True). Under the maintenance transform $U(C_{\text{normal}}) = C_{\text{maint}}$,

$$T\big(A,\, U(C_{\text{normal}})\big) \;=\; \mathbf{1}[0.90 \geq 0.95] \;=\; 0 \quad \text{(False)}.$$

The same observation no longer warrants a page once the context switches to a maintenance window; the truth flips from True to False.

Example 3 (Heightened-risk period tightens escalation (Truthification of the False))**.** *Proposition.* $A :=$ "Escalate to SEV-1 now." *Observable.* capacity shortfall $\sigma = 0.15$ (15%). *Escalation thresholds.* $\kappa_{\text{normal}} = 0.30$ (normal), $\kappa_{\text{risk}} = 0.10$ (heightened-risk period). *Valuation rule.*

$$T(A, C) \;=\; \mathbf{1}\,[\,\sigma \geq \kappa_C\,].$$

Normal context: $T(A, C_{\text{normal}}) = \mathbf{1}[0.15 \geq 0.30] = 0$ (False). After the risk-hardened transform $U(C_{\text{normal}}) = C_{\text{risk}}$,

$$T\big(A,\, U(C_{\text{normal}})\big) \;=\; \mathbf{1}[0.15 \geq 0.10] \;=\; 1 \quad \text{(True)}.$$

A statement that was false in the normal setting becomes true when the organization enters a high-alert period—an instance of *Truthification of the False*.

1.2| Plithogenic Set

A Plithogenic Set [8-10] represents elements by coupling an attribute–value membership with an explicit contradiction measure, thereby extending the formalisms of fuzzy [11], [12], intuitionistic [13], [14], and neutrosophic sets [15], [16].

Definition 4 (Fuzzy Set) ([11], [17]). A *Fuzzy set* τ in a non-empty universe Y is a mapping $\tau : Y \to [0, 1]$. A *fuzzy relation* on Y is a fuzzy subset δ in $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y, then δ is called a *fuzzy relation on* τ if

$$\delta(y, z) \leq \min\{\tau(y), \tau(z)\} \quad \text{for all } y, z \in Y.$$

Definition 5 (Plithogenic Set) ([8], [18]). Let S be a universal set and let $P \subseteq S$ be nonempty. A *Plithogenic Set* is a quintuple

$$PS \;=\; (P,\, v,\, Pv,\, pdf,\, pCF),$$

consisting of:

- an attribute v;
- a set Pv of admissible values of v;
- a *Degree of Appurtenance Function (DAF)* $pdf : P \times Pv \to [0, 1]^s$;[1]
- a *Degree of Contradiction Function (DCF)* $pCF : Pv \times Pv \to [0, 1]^t$.

The DCF satisfies, for all $a, b \in Pv$,

$$\text{Reflexivity: } pCF(a, a) = 0, \qquad \text{Symmetry: } pCF(a, b) = pCF(b, a).$$

Here $s \in \mathbb{N}$ is the appurtenance dimension and $t \in \mathbb{N}$ is the contradiction dimension.

[1] In the literature, DAFs appear in several forms: some are powerset–valued while others take values in the hypercube $[0, 1]^s$. We adopt the latter, classical convention [19].

Example 4 (Change Advisory Board (CAB) evaluation as a Plithogenic Set (general $s \geq 1$))**.** *Universe and subset.* Let S be the set of all proposed production changes for the quarter. Select $P = \{C_A, C_B\} \subseteq S$ as the two candidate changes under review.

Attribute and values. The attribute is $v =$ "impact aspect" with admissible values

$$Pv = \{\text{Availability, Performance, CostEff}\}.$$

DAF (two-dimensional, $s = 2$). For each $(x, a) \in P \times Pv$, define

$$pdf(x, a) = (\text{benefit_fit},\ \text{risk_readiness}) \in [0, 1]^2,$$

encoding how well the change supports aspect a (first coordinate) and how ready it is from an operational-risk standpoint (second coordinate). The assessed values are:

	Availability	Performance	CostEff
C_A	(0.90, 0.60)	(0.80, 0.50)	(0.30, 0.70)
C_B	(0.60, 0.90)	(0.50, 0.80)	(0.70, 0.40)

DCF (scalar, $t = 1$). The Degree of Contradiction between aspect values is specified by

$$pCF(a, b) = pCF(b, a) \in [0, 1], \qquad pCF(a, a) = 0,$$

with the domain-specific antagonisms

$$pCF(\text{Availability}, \text{CostEff}) = 0.70,$$
$$pCF(\text{Performance}, \text{CostEff}) = 0.50,$$
$$pCF(\text{Availability}, \text{Performance}) = 0.20.$$

Interpretation. Availability often trades off against cost efficiency (0.70), performance less so (0.50), while availability and performance are only mildly in tension (0.20). The quintuple $PS = (P, v, Pv, pdf, pCF)$ satisfies reflexivity and symmetry of pCF and provides a plithogenic representation of CAB evidence across aspects and risks.

Definition 6 (Plithogenic Fuzzy Set ($s = 1,\ t = 1$)) ([8], [20-22]). When the appurtenance and contradiction dimensions are both one, a *Plithogenic Fuzzy Set* is a Plithogenic Set $PS = (P, v, Pv, pdf, pCF)$ with

$$pdf : P \times Pv \to [0, 1], \qquad pCF : Pv \times Pv \to [0, 1].$$

For $x \in P$ and $a \in Pv$, set

$$\mu_P(x \mid a) \ :=\ pdf(x, a) \in [0, 1],$$

the (scalar) fuzzy membership of x relative to a. For $a, b \in Pv$, define the scalar contradiction

$$c(a, b) \ :=\ pCF(a, b) \in [0, 1], \qquad c(a, a) = 0,\ \ c(a, b) = c(b, a).$$

Example 5 (Microservice health as a Plithogenic Fuzzy Set ($s = t = 1$))**.** *Universe and subset.* Let S be the set of microservices in an online platform and $P = \{\text{Checkout}\} \subseteq S$ the service under analysis.

Attribute and values. The attribute is $v =$ "operational objective" with

$$Pv = \{\text{HighSLA},\ \text{LowLatency},\ \text{LowCost}\}.$$

DAF (scalar memberships, $s = 1$). Based on recent telemetry and cost reports, define $pdf : P \times Pv \to [0, 1]$ by

$$\mu_P(\text{Checkout} \mid \text{HighSLA}) = 0.90,$$
$$\mu_P(\text{Checkout} \mid \text{LowLatency}) = 0.30,$$
$$\mu_P(\text{Checkout} \mid \text{LowCost}) = 0.60.$$

DCF (scalar contradictions, $t = 1$). Specify

$$c(a, b) = pCF(a, b) = pCF(b, a) \in [0, 1], \qquad c(a, a) = 0,$$

with the operational trade-offs

$$c(\text{LowLatency}, \text{LowCost}) = 0.80,$$

$$c(\text{HighSLA}, \text{LowCost}) = 0.40,$$
$$c(\text{HighSLA}, \text{LowLatency}) = 0.30.$$

Interpretation. The service strongly supports *HighSLA* (0.90), is moderately aligned with *LowCost* (0.60), and currently performs poorly on *LowLatency* (0.30). The largest tension is between low latency and low cost (0.80), reflecting the expected cost of provisioning for speed. The quintuple $PS = (P, v, Pv, pdf, pCF)$ thus realizes a Plithogenic Fuzzy Set ($s = t = 1$) for a system engineer's health evaluation task.

1.3| Upside-Down Logic on a Plithogenic Fuzzy Set with Contradiction Reset

The Upside-Down mechanism with contradiction reset in a Plithogenic Fuzzy Set inverts fuzzy memberships once a contradiction surpasses a chosen threshold and then neutralizes the corresponding contradictions. This yields a consistent representation in contexts where previously conflicting values must be reconciled [23].

Definition 7 (Upside-Down transform with contradiction reset) ([23]). Let $PS = (P, v, Pv, pdf, pCF)$ be a Plithogenic Fuzzy Set and write

$$\mu_P(x \mid a) := pdf(x, a) \in [0, 1], \qquad c(a, b) := pCF(a, b) \in [0, 1],$$

with $c(a, a) = 0$ and $c(a, b) = c(b, a)$. Fix an anchor $b \in Pv$ and a threshold $\tau \in [0, 1]$. Define the *activation set*

$$A_\tau(b) \;:=\; \big\{\, a \in Pv \;:\; c(a, b) \geq \tau \,\big\}.$$

The *Upside-Down* transform produces a new Plithogenic Fuzzy Set

$$PS^{U_{b,\tau}} \;=\; \big(P,\; v,\; Pv,\; pdf^{U_{b,\tau}},\; pCF^{U_{b,\tau}}\big),$$

where

$$pdf^{U_{b,\tau}}(x, a) \;:=\; \begin{cases} 1 - \mu_P(x \mid a), & a \in A_\tau(b), \\ \mu_P(x \mid a), & a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}}$ is given, for all $u, v \in Pv$, by

$$pCF^{U_{b,\tau}}(u, v) \;:=\; \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

In words: if the pair (a, b) activates (i.e., $c(a, b) \geq \tau$), then the membership at a is flipped and the post-transform contradiction between a and the anchor b is *reset to zero*.

Example 6 (Black-Friday latency policy: flip low-cost against a low-latency anchor)**.** *Setting.* Universe $P = \{\text{Checkout}\}$ (the checkout microservice). Single attribute $v =$"objective", with values $Pv = \{\text{LowLatency}(L),\ \text{LowCost}(C),\ \text{HighAvailability}(A)\}$. Let the anchor be $b = L$ (low latency prioritized during the peak-sale window) and choose threshold $\tau = 0.80$.

DAF (memberships). For $x = \text{Checkout}$,

$$\mu_P(x \mid L) = 0.42, \qquad \mu_P(x \mid C) = 0.76, \qquad \mu_P(x \mid A) = 0.88.$$

DCF (contradictions). Symmetric map $c(a, b) = pCF(a, b)$ with

$$c(C, L) = 0.90, \qquad c(A, L) = 0.35, \qquad c(A, C) = 0.40, \qquad c(a, a) = 0.$$

Activation set. With anchor $b = L$ and $\tau = 0.80$,

$$A_\tau(b) = \{\, a \in Pv :\; c(a, L) \geq 0.80 \,\} = \{C\}.$$

Upside-Down transform with contradiction reset. For $a \in A_\tau(b)$, flip the membership and neutralize its contradiction to b:

$$pdf^{U_{b,\tau}}(x, C) = 1 - \mu_P(x \mid C) = 1 - 0.76 = 0.24, \qquad pCF^{U_{b,\tau}}(C, L) = 0.$$

All other values remain:

$$pdf^{U_{b,\tau}}(x, L) = 0.42, \quad pdf^{U_{b,\tau}}(x, A) = 0.88,$$
$$pCF^{U_{b,\tau}}(A, L) = 0.35, \quad pCF^{U_{b,\tau}}(A, C) = 0.40.$$

Interpretation. Under the Black-Friday context, *LowCost* strongly contradicts the *LowLatency* anchor ($0.90 \geq 0.80$), so its membership is inverted (from 0.76 to 0.24) and the *Cost*↔*Latency* tension is cleared by $pCF^{U_{b,\tau}}(C, L) = 0$. Latency and availability memberships are unchanged.

Example 7 (Security incident hardening: flip speed and latency against a strict-security anchor)**.** *Setting.* Universe $P = \{\text{APIGateway}\}$. Single attribute $v =$"operational policy", with values

$$Pv = \{\text{StrictSecurity}(S),\ \text{RapidDeploy}(D),\ \text{LowLatency}(L)\}$$

. During a security incident, the anchor becomes $b = S$ (harden-first), with threshold $\tau = 0.80$.

DAF (memberships). For $x =$ APIGateway,

$$\mu_P(x \mid S) = 0.47, \qquad \mu_P(x \mid D) = 0.81, \qquad \mu_P(x \mid L) = 0.64.$$

DCF (contradictions). Symmetric $c(a, b) = pCF(a, b)$ with

$$c(D, S) = 0.93, \qquad c(L, S) = 0.82, \qquad c(D, L) = 0.30, \qquad c(a, a) = 0.$$

Activation set. With anchor $b = S$ and $\tau = 0.80$,

$$A_\tau(b) = \{\, a \in Pv :\ c(a, S) \geq 0.80 \,\} = \{D, L\}.$$

Upside-Down transform with contradiction reset. Activated values flip and are then neutralized against S:

$$pdf^{U_{b,\tau}}(x, D) = 1 - \mu_P(x \mid D) = 1 - 0.81 = 0.19, \qquad pCF^{U_{b,\tau}}(D, S) = 0,$$
$$pdf^{U_{b,\tau}}(x, L) = 1 - \mu_P(x \mid L) = 1 - 0.64 = 0.36, \qquad pCF^{U_{b,\tau}}(L, S) = 0.$$

Non-activated values remain:

$$pdf^{U_{b,\tau}}(x, S) = 0.47, \qquad pCF^{U_{b,\tau}}(D, L) = 0.30.$$

Interpretation. Entering an incident-hardened context flips the memberships of *RapidDeploy* and *LowLatency* (both highly contradictory to *StrictSecurity*) from $(0.81, 0.64)$ to $(0.19, 0.36)$, and removes their pairwise contradictions with the security anchor. This yields a consistent, security-first representation for subsequent reasoning.

1.4| Two-Mode De-Plithogenication

We now formalize *Two-mode De-Plithogenication.* The mechanism covers two distinct situations: 1) a contradiction that reflects a *real* conflict, and 2) a contradiction that is merely *apparent* and should not cause an inversion [23].

Definition 8 (Improved De-Plithogenication (two-mode; general plithogenic set)) ([23]). Let $PS = (P, v, Pv, \text{pdf}, \text{pCF})$ be a plithogenic set with $\text{pdf} : P \times Pv \to [0, 1]$ the Degree of Appurtenance Function (DAF) and $\text{pCF} : Pv \times Pv \to [0, 1]$ the Degree of Contradiction Function (DCF), symmetric and satisfying $\text{pCF}(a, a) = 0$. Fix an *anchor* $b \in Pv$ and a *threshold* $\tau \in [0, 1]$.

Activation. For $a \in Pv$, declare that *a activates (relative to* b, τ*)* when

$$\mathsf{Act}(a \mid b, \tau) \ :=\ \mathbf{1}\left[\,\text{pCF}(a, b) \ \geq\ \tau\,\right] = 1.$$

Mode selection. Let $\mathcal{M} : Pv \times Pv \to \{0, 1\}$ be a (policy) *mode selector* on unordered pairs, with $\mathcal{M}(\{a, b\}) = \mathcal{M}(\{b, a\})$. For any activated pair $\{a, b\}$, the semantics are:

- Mode 0 (Neutralize-only): $\mathcal{M}(\{a, b\}) = 0$ means the contradiction is only apparent. No appurtenance coordinate associated with a (or b) is flipped; the contradiction is simply reset to 0.

- Mode 1 (Invert+Neutralize): $\mathcal{M}(\{a,b\}) = 1$ means the contradiction is genuine. We first invert the appurtenance coordinate(s) tied to a (or b) and then reset their contradiction to 0.

Two-mode Upside-Down operator. Define $U^{\mathrm{imp}}_{b,\tau,\mathcal{M}}(PS) =: (P, v, Pv, \mathrm{pdf}^U, \mathrm{pCF}^U)$ by

$$\mathrm{pdf}^U(x,a) := \begin{cases} 1-\mathrm{pdf}(x,a), & \mathsf{Act}(a \mid b,\tau) = 1 \text{ and } \mathcal{M}(\{a,b\}) = 1, \\ \mathrm{pdf}(x,a), & \text{otherwise}, \end{cases} \qquad (x \in P,\ a \in Pv),$$

$$\mathrm{pCF}^U(u,w) := \begin{cases} 0, & \mathsf{Act}(u \mid b,\tau) = 1 \text{ or } \mathsf{Act}(w \mid b,\tau) = 1, \\ \mathrm{pCF}(u,w), & \text{otherwise}, \end{cases} \qquad (u,w \in Pv, \text{ with symmetry preserved}).$$

Two-mode De-Plithogenication sequence. A finite composition

$$PS^{\text{imp-dep}} := U^{\mathrm{imp}}_{b_k,\tau_k,\mathcal{M}_k} \circ \cdots \circ U^{\mathrm{imp}}_{b_2,\tau_2,\mathcal{M}_2} \circ U^{\mathrm{imp}}_{b_1,\tau_1,\mathcal{M}_1}(PS)$$

is called an *Improved De-Plithogenication* when, for every pair $\{u,w\} \subseteq Pv$ that activates at least once in the sequence, the final contradiction vanishes: $\mathrm{pCF}^{\text{imp-dep}}(u,w) = 0$. If the activated pairs cover all of $Pv \times Pv$, then $\mathrm{pCF}^{\text{imp-dep}} \equiv 0$. Moreover, once this stabilized state is reached, the operator acts idempotently.

Example 8 (Peak-traffic latency policy: Mode 1 for throughput, Mode 0 for cost)**.** *Setting.* Universe $P = \{\text{Checkout}\}$ (checkout service). Single attribute $v =$"objective", with values $Pv = \{L = \text{LowLatency},\ T = \text{HighThroughput},\ C = \text{CostEfficiency}\}$. During a flash sale, the anchor is $b = L$ (latency first). Threshold $\tau = 0.80$.

DAF (memberships). For $x = \text{Checkout}$,

$$\mu_P(x \mid L) = 0.46, \qquad \mu_P(x \mid T) = 0.62, \qquad \mu_P(x \mid C) = 0.78.$$

DCF to the anchor. Symmetric contradictions $c(\cdot,\cdot) = pCF(\cdot,\cdot)$:

$$c(T,L) = 0.90 \quad (\text{genuine conflict} \Rightarrow \mathcal{M}(\{T,L\}) = 1),$$
$$c(C,L) = 0.85 \quad (\text{apparent conflict} \Rightarrow \mathcal{M}(\{C,L\}) = 0),$$

and $c(a,a) = 0$. Both pairs activate since $c(\cdot,L) \geq \tau$.

Apply the two-mode operator $U^{\mathrm{imp}}_{b,\tau,\mathcal{M}}$.

$$\mathrm{pdf}^U(x,T) = 1 - \mathrm{pdf}(x,T) = 1 - 0.62 = 0.38 \quad (\text{flip, Mode 1}),$$
$$\mathrm{pdf}^U(x,C) = \mathrm{pdf}(x,C) = 0.78 \quad (\text{no flip, Mode 0}),$$
$$\mathrm{pdf}^U(x,L) = \mathrm{pdf}(x,L) = 0.46.$$

Neutralization (for any activated pair with L):

$$\mathrm{pCF}^U(T,L) = 0, \qquad \mathrm{pCF}^U(C,L) = 0.$$

Interpretation. High throughput genuinely contradicts the latency-first anchor and is inverted (penalized), while the cost tension is policy-approved (apparent), so its membership is retained. All activated tensions to the anchor are cleared to 0, preventing double-counting of conflicts.

Example 9 (Security hardening window: flip speed/latency, keep audit (apparent), then neutralize)**.** *Setting.* Universe $P = \{\text{APIGateway}\}$. Single attribute $v =$"operational policy", with values $Pv = \{S = \text{StrictSecurity},\ D = \text{RapidDeploy},\ L = \text{LowLatency},\ A = \text{AuditCoverage}\}$. In an incident-hardening window, the anchor is $b = S$. Threshold $\tau = 0.80$.

DAF (memberships). For $x = \text{APIGateway}$,

$$\mu_P(x \mid S) = 0.52, \qquad \mu_P(x \mid D) = 0.83, \qquad \mu_P(x \mid L) = 0.67, \qquad \mu_P(x \mid A) = 0.58.$$

DCF to the anchor and mode choices.

$$c(D,S) = 0.92 \quad (\text{genuine} \Rightarrow \mathcal{M}(\{D,S\}) = 1),$$

$$c(L,S) = 0.86 \quad (\text{genuine} \Rightarrow \mathcal{M}(\{L,S\}) = 1),$$
$$c(A,S) = 0.81 \quad (\text{apparent} \Rightarrow \mathcal{M}(\{A,S\}) = 0),$$

with $c(a,a) = 0$. All three activate since $c(\cdot, S) \geq \tau$.

Apply the two-mode operator $U^{\text{imp}}_{b,\tau,\mathcal{M}}$.

$$\text{pdf}^U(x,D) = 1 - \text{pdf}(x,D) = 1 - 0.83 = 0.17 \quad (\text{flip, Mode 1}),$$
$$\text{pdf}^U(x,L) = 1 - \text{pdf}(x,L) = 1 - 0.67 = 0.33 \quad (\text{flip, Mode 1}),$$
$$\text{pdf}^U(x,A) = \text{pdf}(x,A) = 0.58 \quad (\text{no flip, Mode 0}),$$
$$\text{pdf}^U(x,S) = \text{pdf}(x,S) = 0.52.$$

Neutralize all activated contradictions to the anchor:

$$\text{pCF}^U(D,S) = 0, \qquad \text{pCF}^U(L,S) = 0, \qquad \text{pCF}^U(A,S) = 0.$$

Interpretation. Under security-first posture, *RapidDeploy* and *LowLatency* are inverted as genuine conflicts with *StrictSecurity*. *AuditCoverage* is deemed a false-positive conflict (e.g., tool artifacts), so it is preserved. In all cases, the post-transform structure removes residual tension with the anchor, yielding a consistent basis for downstream scoring.

1.5| Fuzzy ITIL

ITIL (Information Technology Infrastructure Library) is a framework of best practices guiding IT service management, emphasizing service lifecycle, efficiency, alignment, and continual improvement [24-27]. A Fuzzy ITIL system applies fuzzy logic to ITIL lifecycle KPIs, enabling graded assessments of service maturity under uncertainty[28], [29].

Definition 9 (Fuzzy ITIL (FITIL)) **(**[29]). Let

$$\mathcal{L} := \{\text{SS}, \text{SD}, \text{ST}, \text{SO}, \text{CSI}\}$$

denote the ITIL life-cycle areas (Service Strategy, Service Design, Service Transition, Service Operation, Continual Service Improvement). For each $\ell \in \mathcal{L}$, fix a finite KPI index set $K(\ell) = \{1, \ldots, m_\ell\}$ and let $x_{\ell,k} \in D_{\ell,k} \subseteq \mathbb{R}$ be the measured value of KPI k in area ℓ.

A *Fuzzy ITIL system* is the tuple

$$\text{FITIL} = \Big(\mathcal{L},\ \{D_{\ell,k}\},\ \mathcal{T},\ \mu,\ R,\ \otimes,\ \oplus,\ \text{Defz},\ \{z_r\}_{r\in R},\ \{\alpha_\ell\}_{\ell\in\mathcal{L}}\Big),$$

with the following components.

(i) **Linguistic terms.** $\mathcal{T}$ is a finite term set (e.g. $\{\text{Low}, \text{Medium}, \text{High}\}$) used to describe KPI quality states and maturity outputs.

(ii) **Memberships.** For each KPI domain $D_{\ell,k}$ and term $t \in \mathcal{T}$, a membership function

$$\mu_{\ell,k,t} : D_{\ell,k} \to [0,1]$$

is given. In applications we often choose triangular fuzzy numbers. Writing the triangle for term t as (a,b,c) with $a < b < c$,

$$\mu_{(a,b,c)}(x) = \begin{cases} 0, & x \leq a \text{ or } x \geq c, \\ \dfrac{x-a}{b-a}, & a < x \leq b, \\ \dfrac{c-x}{c-b}, & b < x < c. \end{cases}$$

(Other shapes—trapezoidal, Gaussian—are admissible.)

(iii) **Rule base.** R is a finite set of (Sugeno-type) rules indexed by r:

$$\text{Rule } r: \quad \bigwedge_{(\ell,k)} \big(x_{\ell,k} \text{ is } t_{\ell,k}^{(r)}\big) \implies \text{Maturity is } z_r,$$

where $t_{\ell,k}^{(r)} \in \mathcal{T}$ and $z_r \in [1,5]$ is a constant consequent representing the (crisp) maturity level recommended by the rule (1=Initial, 2=Repeatable, 3=Defined, 4=Managed, 5=Optimized).

(iv) **Inference/aggregation.** For an observation $x = \{x_{\ell,k}\}$, the firing strength of rule r is

$$w_r(x) := \bigotimes_{(\ell,k)} \mu_{\ell,k,t_{\ell,k}^{(r)}}\big(x_{\ell,k}\big),$$

where $\otimes$ is a t-norm (e.g. min or product). The *area-level* fuzzy maturity (Sugeno weighted average) is

$$M_\ell(x) := \frac{\sum_{r\in R_\ell} w_r(x)\, z_r}{\sum_{r\in R_\ell} w_r(x)},$$

with R_ℓ the subset of rules whose antecedents mention only KPIs from area ℓ. If desired, a *global* maturity is a convex blend

$$M_{\text{global}}(x) := \sum_{\ell\in\mathcal{L}} \alpha_\ell\, M_\ell(x), \qquad \alpha_\ell \ge 0, \ \sum_\ell \alpha_\ell = 1.$$

For Mamdani-style consequents (term-valued), replace $\{z_r\}$ by output membership functions and let $\oplus$ be an s-norm (e.g. max) and Defz the centroid; both styles are compatible with FITIL.

(v) **Defuzzification.** In the Sugeno form above, Defz is already the weighted average; in Mamdani form use the centroid over the aggregated output set.

Example 10 (24/7 online retail platform—Incident Management in Service Operation)**.** We compute a concrete FITIL maturity for the SO area from three KPIs measured over the last month:

$$\text{SLA compliance } s = 0.92, \qquad \text{MTTR } m = 3.5 \text{ hours}, \qquad \text{Incident backlog } b = 12 \text{ tickets}.$$

Define the following triangular memberships (all values are dimensionless after scaling to the stated units).

$$\text{SLA High } H_s : (0.85,\ 0.95,\ 1.00), \qquad \text{SLA Medium } M_s : (0.75,\ 0.85,\ 0.95);$$
$$\text{MTTR Low } L_m : (0,\ 2,\ 4), \qquad \text{MTTR Medium } M_m : (3,\ 5,\ 7);$$
$$\text{Backlog Low } L_b : (0,\ 5,\ 15), \qquad \text{Backlog Medium } M_b : (10,\ 20,\ 30).$$

Fuzzification yields (compute each by the triangular formula):

$$\mu_{H_s}(0.92) = \frac{0.92-0.85}{0.95-0.85} = \frac{0.07}{0.10} = 0.70, \quad \mu_{M_s}(0.92) = \frac{0.95-0.92}{0.95-0.85} = \frac{0.03}{0.10} = 0.30;$$
$$\mu_{L_m}(3.5) = \frac{4-3.5}{4-2} = \frac{0.5}{2} = 0.25, \quad \mu_{M_m}(3.5) = \frac{3.5-3}{5-3} = \frac{0.5}{2} = 0.25;$$
$$\mu_{L_b}(12) = \frac{15-12}{15-5} = \frac{3}{10} = 0.30, \quad \mu_{M_b}(12) = \frac{12-10}{20-10} = \frac{2}{10} = 0.20.$$

Use a Sugeno rule base with $\otimes = \min$ and constants $z_r \in \{3.0, 3.5, 4.5\}$ (on the 1–5 ITIL scale):

R1 If (SLA is High) $\wedge$ (MTTR is Low) $\wedge$ (Backlog is Low) then $z_1 = 4.5$ (Managed-high).

R2 If (SLA is Medium) $\wedge$ $\big($(MTTR is Medium) $\vee$ (Backlog is Medium)$\big)$ then $z_2 = 3.0$ (Defined).

R3 If (SLA is High) $\wedge$ $\big($(MTTR is Medium) $\vee$ (Backlog is Medium)$\big)$ then $z_3 = 3.5$ (Upper-Defined).

With $\vee$ implemented by max and $\wedge$ by min, the firing strengths are:

$$w_1 = \min(0.70,\ 0.25,\ 0.30) = 0.25, \qquad w_2 = \min\big(0.30,\ \max(0.25, 0.20)\big) = \min(0.30, 0.25) = 0.25,$$
$$w_3 = \min\big(0.70,\ \max(0.25, 0.20)\big) = \min(0.70, 0.25) = 0.25.$$

The SO maturity by Sugeno defuzzification is the weighted average

$$M_{\text{SO}} = \frac{w_1 z_1 + w_2 z_2 + w_3 z_3}{w_1 + w_2 + w_3} = \frac{0.25\cdot 4.5 + 0.25\cdot 3.0 + 0.25\cdot 3.5}{0.75} = \frac{1.125 + 0.750 + 0.875}{0.75} = \frac{2.750}{0.75} = 3.\overline{6}\,.$$

Thus the observed SO process is between *Defined* (3) and *Managed* (4), trending towards 4. On a full FITIL deployment, one would define analogous rule blocks for SS/SD/ST/CSI and combine them as

$$M_{\text{global}} = \sum_{\ell \in \{\text{SS,SD,ST,SO,CSI}\}} \alpha_\ell \, M_\ell,$$

e.g. with $\alpha_\ell = \frac{1}{5}$ if all areas are equally weighted.

Proposition 1 (Crisp reduction (degeneracy))**.** *Suppose for all (ℓ, k) and all terms t the memberships are* crisp *indicators (i.e. $\mu_{\ell,k,t} \in \{0, 1\}$) and that every observation activates exactly one term per KPI. Then, for either Sugeno or Mamdani inference, $M_\ell(x)$ coincides with the corresponding crisp maturity scoring used in standard ITIL assessments (e.g. level 1–5 by questionnaire mapping).*

Proof: In the Sugeno case, exactly one rule per KPI-term pattern has $w_r = 1$ and all others have $w_r = 0$, hence $M_\ell = z_r$ is the crisp mapped score. In the Mamdani case the aggregated output reduces to a single consequent set whose centroid is the crisp level. This matches the FSIM-style degeneracy where fuzzy numbers collapse to ordinary scalars. □

Proposition 2 (Monotonicity under KPI improvement)**.** *Fix a rule base in which consequents are monotone with respect to "good" terms. If a change in measurements increases (does not decrease) the memberships of all "good" terms and decreases (does not increase) those of all "bad" terms, then $M_\ell(x)$ is non-decreasing (for $\otimes = \min$ or product and Sugeno defuzzification).*

Proof: For $\otimes = \min$ or product, each w_r is monotone in its antecedent memberships. A monotone shift from bad to good increases the weights of higher-z_r rules and decreases those of lower-z_r rules; the weighted average therefore cannot decrease. □

2| Main Results

In this section, we examine the results of this paper.

2.1| Plithogenic Fuzzy ITIL

Plithogenic Fuzzy ITIL extends fuzzy ITIL by adding contradiction functions among attributes, capturing conflicting service objectives within ITIL maturity evaluation.

Notation 3. *Let*

$$\mathcal{L} = \{\text{SS}, \text{SD}, \text{ST}, \text{SO}, \text{CSI}\}$$

be the ITIL life-cycle areas. For each $\ell \in \mathcal{L}$ fix KPI indices $K(\ell) = \{1, \ldots, m_\ell\}$ and domains $D_{\ell,k} \subseteq \mathbb{R}$ with observed values $x_{\ell,k} \in D_{\ell,k}$. Let $\mathcal{I} := \{(\ell, k) \mid \ell \in \mathcal{L},\, k \in K(\ell)\}$ denote the set of KPI instances.

A finite set of linguistic terms $\mathcal{T}$ (e.g. $\{Low, Medium, High\}$) is given for each KPI.[2] *For each $(\ell, k) \in \mathcal{I}$ and $t \in \mathcal{T}$, let $\mu_{\ell,k,t} : D_{\ell,k} \to [0, 1]$ be a fuzzy membership (e.g. triangular/trapezoidal).*

We now endow FITIL with a plithogenic layer by introducing a *contradiction function* between attribute values (linguistic terms) and by adjusting memberships accordingly.

Definition 10 (PFITIL attribute space)**.** Define a single (composite) attribute v whose values record the KPI instance and a linguistic term:

$$Pv \;:=\; \mathcal{I} \times \mathcal{T}.$$

We write $a = ((\ell, k), t) \in Pv$. The *Degree of Contradiction Function (DCF)* is

$$pCF : Pv \times Pv \longrightarrow [0, 1], \qquad pCF(a, b) = pCF(b, a), \quad pCF(a, a) = 0.$$

[2]If needed, one may allow KPI-specific term sets $\mathcal{T}_{\ell,k}$ and embed them into a disjoint union. For clarity we present the uniform-$\mathcal{T}$ case; the generalization is straightforward.

Typically pCF encodes domain antagonisms (e.g. *Low MTTR* vs. *High MTTR*, *High SLA* vs. *Low Cost*, etc.).

Example 11 (PFITIL attribute space for SO/SS KPIs with domain antagonisms)**.** *Setup.* Let the KPI-instance index be

$$\mathcal{I} = \{(\mathrm{SO}, \mathrm{MTTR}),\ (\mathrm{SO}, \mathrm{SLA}),\ (\mathrm{SS}, \mathrm{Cost})\},$$

and the common linguistic term set be $\mathcal{T} = \{\mathrm{Low}, \mathrm{Medium}, \mathrm{High}\}$. The composite attribute-value space is

$$Pv = \mathcal{I} \times \mathcal{T}, \quad a = \big((\ell, k), t\big) \in Pv.$$

Degree of Contradiction Function. Define $pCF : Pv \times Pv \to [0, 1]$ symmetric with $pCF(a, a) = 0$, capturing typical trade-offs:

$$pCF\big(((\mathrm{SO}, \mathrm{MTTR}), \mathrm{Low}), ((\mathrm{SO}, \mathrm{MTTR}), \mathrm{High})\big) = 1.00,$$
$$pCF\big(((\mathrm{SO}, \mathrm{SLA}), \mathrm{High}), ((\mathrm{SS}, \mathrm{Cost}), \mathrm{Low})\big) = 0.80,$$
$$pCF\big(((\mathrm{SO}, \mathrm{SLA}), \mathrm{High}), ((\mathrm{SO}, \mathrm{MTTR}), \mathrm{Low})\big) = 0.10,$$
$$pCF\big(((\mathrm{SS}, \mathrm{Cost}), \mathrm{Low}), ((\mathrm{SO}, \mathrm{MTTR}), \mathrm{Low})\big) = 0.30.$$

Reading. Very low MTTR conflicts maximally with high MTTR; high SLA often opposes low cost; SLA and low MTTR align (small contradiction); and low cost mildly conflicts with low MTTR due to extra capacity needs.

Definition 11 (Underlying universe and DAF)**.** Let the universe be the KPI-instance set $P := \mathcal{I}$ (elements are identifiers $u = (\ell, k)$). Fix the observation vector $x := \{x_{\ell,k}\}_{(\ell,k)\in\mathcal{I}}$. For an *anchor map* $b : \mathcal{I} \to \mathcal{T}$ marking each KPI's *target term* (e.g. $b(\mathrm{SLA}) = \mathrm{High}$, $b(\mathrm{MTTR}) = \mathrm{Low}$), define the *plithogenic adjusted membership* (DAF)

$$pdf^{\mathrm{pl}} : P \times Pv \to [0, 1],$$

$$pdf^{\mathrm{pl}}\big(u = ((\ell, k)),\ ((\ell, k), t)\big)$$

$$:=\ (1 - c_{\ell,k,t})\, \mu_{\ell,k,t}(x_{\ell,k})\ +\ c_{\ell,k,t}\, \big(1 - \mu_{\ell,k,t}(x_{\ell,k})\big),$$

where the *local contradiction* uses the chosen anchor

$$c_{\ell,k,t}\ :=\ pCF\Big(\big((\ell, k), t\big),\ \big((\ell, k), b(\ell, k)\big)\Big)\ \in [0, 1].$$

For $a = ((\ell', k'), t')$ with $(\ell', k') \neq (\ell, k)$, set $pdf^{\mathrm{pl}}((\ell, k), a) := 0$ (attributes are KPI-local).

Remark 1 (Closed form and basic bounds)**.** For brevity write $\mu := \mu_{\ell,k,t}(x_{\ell,k})$ and $c := c_{\ell,k,t}$. Then

$$pdf^{\mathrm{pl}} = (1 - c)\mu + c(1 - \mu) = \mu + c(1 - 2\mu),$$

a convex combination of μ and $1 - \mu$; hence $0 \le pdf^{\mathrm{pl}} \le 1$. Moreover,

$$\frac{\partial\, pdf^{\mathrm{pl}}}{\partial c} = 1 - 2\mu.$$

Thus, for *good* terms with $\mu \ge \frac{1}{2}$, increasing contradiction lowers membership; for *bad* terms with $\mu \le \frac{1}{2}$, increasing contradiction raises membership (penalization vs. inversion).

Example 12 (Underlying universe and plithogenic DAF with anchors and explicit calculations)**.** *Universe and observations.* Let $P := \mathcal{I} = \{(\mathrm{SO}, \mathrm{MTTR}), (\mathrm{SO}, \mathrm{SLA}), (\mathrm{SS}, \mathrm{Cost})\}$. Assume measurements

$$x_{\mathrm{SO,MTTR}} = 3.5 \text{ h},$$

$$x_{\mathrm{SO,SLA}} = 0.92,$$

$$x_{\mathrm{SS,Cost}} = 0.012 \text{ USD/request}.$$

Use triangular memberships (a,b,c) with the standard formula $\mu_{(a,b,c)}(x) = \frac{x-a}{b-a}$ for $a < x \le b$ and $\mu_{(a,b,c)}(x) = \frac{c-x}{c-b}$ for $b < x < c$.

$$\text{MTTR Low } (0,2,4): \ \mu_L(3.5) = \frac{4-3.5}{4-2} = 0.25,$$
$$\text{MTTR Med } (3,5,7): \ \mu_M(3.5) = \frac{3.5-3}{5-3} = 0.25;$$
$$\text{SLA High } (0.85,0.95,1.00): \ \mu_H(0.92) = \frac{0.92-0.85}{0.95-0.85} = 0.70,$$
$$\text{SLA Med } (0.75,0.85,0.95): \ \mu_M(0.92) = \frac{0.95-0.92}{0.95-0.85} = 0.30;$$
$$\text{Cost Low } (0.005,0.010,0.020): \ \mu_L(0.012) = \frac{0.020-0.012}{0.020-0.010} = 0.80,$$
$$\text{Cost Med } (0.010,0.020,0.030): \ \mu_M(0.012) = \frac{0.012-0.010}{0.020-0.010} = 0.20.$$

Anchor map and contradictions. Let $b(\text{MTTR}) = \text{Low}$, $b(\text{SLA}) = \text{High}$, $b(\text{Cost}) = \text{Low}$. Specify local DCFs against each KPI's anchor:

$$c_{\text{SO,MTTR,Low}} = pCF(((\text{SO},\text{MTTR}),\text{Low}),((\text{SO},\text{MTTR}),\text{Low})) = 0,$$
$$c_{\text{SO,MTTR,Med}} = pCF(((\text{SO},\text{MTTR}),\text{Med}),((\text{SO},\text{MTTR}),\text{Low})) = 0.60;$$
$$c_{\text{SO,SLA,High}} = 0, \qquad c_{\text{SO,SLA,Med}} = 0.50;$$
$$c_{\text{SS,Cost,Low}} = 0, \qquad c_{\text{SS,Cost,Med}} = 0.70.$$

Adjusted (plithogenic) memberships. For $u = ((\ell,k))$ and $a = ((\ell,k),t)$,

$$pdf^{\text{pl}}(u,a) = (1 - c_{\ell,k,t})\,\mu_{\ell,k,t}(x_{\ell,k}) + c_{\ell,k,t}\big(1 - \mu_{\ell,k,t}(x_{\ell,k})\big).$$

Compute explicitly:

$$pdf^{\text{pl}}((\text{SO},\text{MTTR}),((\text{SO},\text{MTTR}),\text{Low})) = (1-0)\cdot 0.25 + 0\cdot 0.75 = 0.25,$$
$$pdf^{\text{pl}}((\text{SO},\text{MTTR}),((\text{SO},\text{MTTR}),\text{Med})) = (1-0.60)\cdot 0.25 + 0.60\cdot 0.75 = 0.10 + 0.45 = 0.55;$$
$$pdf^{\text{pl}}((\text{SO},\text{SLA}),((\text{SO},\text{SLA}),\text{High})) = (1-0)\cdot 0.70 + 0\cdot 0.30 = 0.70,$$
$$pdf^{\text{pl}}((\text{SO},\text{SLA}),((\text{SO},\text{SLA}),\text{Med})) = (1-0.50)\cdot 0.30 + 0.50\cdot 0.70 = 0.15 + 0.35 = 0.50;$$
$$pdf^{\text{pl}}((\text{SS},\text{Cost}),((\text{SS},\text{Cost}),\text{Low})) = (1-0)\cdot 0.80 = 0.80,$$
$$pdf^{\text{pl}}((\text{SS},\text{Cost}),((\text{SS},\text{Cost}),\text{Med})) = (1-0.70)\cdot 0.20 + 0.70\cdot 0.80 = 0.06 + 0.56 = 0.62.$$

(For KPI-locality, $pdf^{\text{pl}}((\text{SO},\text{MTTR}),((\text{SO},\text{SLA}),t)) = 0$, etc.)

Definition 12 (PFITIL rule firing and maturity)**.** Let R be a set of Sugeno-type rules indexed by r:

$$\text{Rule } r: \quad \bigwedge_{(\ell,k)\in A_r} \big((\ell,k) \text{ is } t^{(r)}_{\ell,k}\big) \implies z_r \in [1,5].$$

Denote the antecedent attribute values by $a^{(r)}_{\ell,k} := ((\ell,k), t^{(r)}_{\ell,k})$ and their adjusted memberships by $\mu^{\text{pl}}_{\ell,k,r} := pdf^{\text{pl}}\big((\ell,k), a^{(r)}_{\ell,k}\big)$. With a fixed t-norm T (e.g. min or product), the rule firing strength is

$$w^{\text{pl}}_r \ := \ T\big(\{\mu^{\text{pl}}_{\ell,k,r}\}_{(\ell,k)\in A_r}\big).$$

For each area ℓ, let $R_\ell \subseteq R$ be rules involving only KPIs from ℓ. The *PFITIL area maturity* is the Sugeno weighted average

$$M^{\text{pl}}_\ell \ := \ \frac{\sum_{r\in R_\ell} w^{\text{pl}}_r\, z_r}{\sum_{r\in R_\ell} w^{\text{pl}}_r},$$

and the *global maturity* is the convex blend $M^{\text{pl}}_{\text{global}} := \sum_\ell \alpha_\ell M^{\text{pl}}_\ell$ with $\alpha_\ell \ge 0$, $\sum_\ell \alpha_\ell = 1$.

Example 13 (PFITIL rule firing and maturity (Sugeno, $T = \min$) for Service Operation)**.** *Area and rules.* Consider $\ell = \mathrm{SO}$ with KPIs $(\mathrm{SO}, \mathrm{SLA})$ and $(\mathrm{SO}, \mathrm{MTTR})$. Use the adjusted memberships from the previous example:

$$\mu^{\mathrm{pl}}_{\mathrm{SLA,High}} = 0.70, \quad \mu^{\mathrm{pl}}_{\mathrm{SLA,Med}} = 0.50,$$
$$\mu^{\mathrm{pl}}_{\mathrm{MTTR,Low}} = 0.25, \quad \mu^{\mathrm{pl}}_{\mathrm{MTTR,Med}} = 0.55.$$

Two Sugeno-type rules with consequents on the 1–5 scale:

$$\textbf{R1:}\ (\text{SLA is High}) \wedge (\text{MTTR is Low}) \implies z_1 = 4.5,$$
$$\textbf{R2:}\ (\text{SLA is Medium}) \wedge (\text{MTTR is Medium}) \implies z_2 = 3.0.$$

Firing strengths ($T = \min$).

$$w^{\mathrm{pl}}_1 = \min(0.70,\, 0.25) = 0.25, \qquad w^{\mathrm{pl}}_2 = \min(0.50,\, 0.55) = 0.50.$$

Area maturity (Sugeno weighted average).

$$M^{\mathrm{pl}}_{\mathrm{SO}} = \frac{w^{\mathrm{pl}}_1 z_1 + w^{\mathrm{pl}}_2 z_2}{w^{\mathrm{pl}}_1 + w^{\mathrm{pl}}_2} = \frac{0.25 \cdot 4.5 + 0.50 \cdot 3.0}{0.25 + 0.50} = \frac{1.125 + 1.500}{0.75} = \frac{2.625}{0.75} = 3.5.$$

Reading. Under the observed data and contradictions, the SO maturity lands midway between "Defined" (3) and "Managed" (4).

Proposition 3 (Monotonicity under KPI improvements)**.** *Fix the rule base and anchors $b(\ell, k)$. If for a given antecedent term t the* raw *memberships $\mu_{\ell,k,t}(x_{\ell,k})$ weakly increase while the contradictions $c_{\ell,k,t}$ do not increase, then every w^{pl}_r is non-decreasing (for $T = \min$ or product), and hence each M^{pl}_ℓ is non-decreasing.*

Proof: Each adjusted membership is $\mu^{\mathrm{pl}} = (1 - c)\mu + c(1 - \mu)$, which is non-decreasing in μ when $c \le \frac{1}{2}$ and still non-decreasing in μ at fixed c for $T \in \{\min,\ \text{product}\}$ because T is monotone in each argument. Non-increasing c also weakly increases μ^{pl} whenever $\mu \ge \frac{1}{2}$ (the typical case for "good" terms anchored by b). Thus every $w^{\mathrm{pl}}_r = T(\cdots)$ cannot decrease, and the Sugeno averages inherit the monotonicity. □

Proposition 4 (Crisp and fuzzy degeneracy)**.** *1) If $pCF \equiv 0$ then PFITIL reduces to FITIL (Theorem 1). 2) If, in addition, each $\mu_{\ell,k,t}$ is $\{0, 1\}$-valued and exactly one term per KPI is 1, then M^{pl}_ℓ coincides with the corresponding crisp ITIL scoring.*

Proof: Part (i) is Theorem 1. For (ii), under single-term activation, each rule weight is either 0 or 1 exactly as in the crisp mapping, so the Sugeno average returns the crisp level. □

Theorem 1 (PFITIL ⇒ FITIL by zero contradiction)**.** *Assume $pCF \equiv 0$ on $Pv \times Pv$ (hence $c_{\ell,k,t} = 0$ for all $(\ell, k), t)$. Then for any fixed rule base R, t-norm T, and weights $\{\alpha_\ell\}$,*

$$M^{\mathrm{pl}}_\ell = M_\ell \quad \textit{and} \quad M^{\mathrm{pl}}_{\mathrm{global}} = M_{\mathrm{global}},$$

i.e. PFITIL exactly reduces to the FITIL pipeline.

Proof: If $pCF \equiv 0$, then $pdf^{\mathrm{pl}}\big((\ell, k), ((\ell, k), t)\big) = (1 - 0)\mu_{\ell,k,t}(x_{\ell,k}) + 0 \cdot (1 - \mu_{\ell,k,t}(x_{\ell,k})) = \mu_{\ell,k,t}(x_{\ell,k})$. Hence each antecedent membership equals its FITIL counterpart, so $w^{\mathrm{pl}}_r = w_r$. Identical rule weights and consequents imply identical Sugeno averages at area and global levels. □

Theorem 2 (Structural theorem: PFITIL is a Plithogenic Fuzzy Set)**.** *The quintuple*

$$\big(P,\ v,\ Pv,\ pdf^{\mathrm{pl}},\ pCF\big)$$

from Definitions 2.2–2.7 is a Plithogenic Fuzzy Set with appurtenance and contradiction dimensions $s = t = 1$.

Proof: By Definition 10, v is a single attribute with value set $Pv = \mathcal{I} \times \mathcal{T}$, so the attribute component complies with the plithogenic schema. The DCF $pCF : Pv \times Pv \to [0,1]$ is specified to be symmetric with $pCF(a,a) = 0$, satisfying the required reflexivity and symmetry axioms.

For the DAF, $pdf^{\mathrm{pl}} : P \times Pv \to [0,1]$ is well-defined by Definition 11. For each KPI instance $u = (\ell, k)$ and term t, the closed form

$$pdf^{\mathrm{pl}} = (1-c)\mu + c(1-\mu), \qquad \mu := \mu_{\ell,k,t}(x_{\ell,k}),\ c := pCF\big(((\ell,k),t),((\ell,k),b(\ell,k))\big)$$

is a convex combination of μ and $1-\mu$, thus maps into $[0,1]$. Therefore $(P, v, Pv, pdf^{\mathrm{pl}}, pCF)$ obeys the Plithogenic Fuzzy Set definition with $s = 1$, $t = 1$. □

2.2| Upside-Down Logic in Plithogenic Fuzzy ITIL

Upside-Down Logic in PFITIL flips fuzzy memberships when terms contradict context anchors, then resets contradictions, enabling reversible, context-aware KPI reasoning and coherent ITIL maturity decisions.

Notation 4. *Let $\mathcal{L} = \{\mathrm{SS}, \mathrm{SD}, \mathrm{ST}, \mathrm{SO}, \mathrm{CSI}\}$ be the ITIL areas and $\mathcal{I} := \{(\ell,k) \mid \ell \in \mathcal{L},\ k \in \{1,\dots,m_\ell\}\}$ the KPI instances with measurements $x_{\ell,k} \in D_{\ell,k} \subseteq \mathbb{R}$. Let $\mathcal{T}$ be a finite term set (e.g.* $\{\mathrm{Low}, \mathrm{Medium}, \mathrm{High}\}$*). We use the plithogenic attribute/value space*

$$v \text{ (single attribute)}, \qquad Pv := \mathcal{I} \times \mathcal{T}, \qquad a = ((\ell,k),t) \in Pv.$$

A Degree of Contradiction Function (DCF) is given:

$$pCF : Pv \times Pv \to [0,1], \quad pCF(a,b) = pCF(b,a), \quad pCF(a,a) = 0.$$

A (context-free) fuzzy membership $\mu_{\ell,k,t} : D_{\ell,k} \to [0,1]$ yields the DAF $pdf((\ell,k),((\ell,k),t)) := \mu_{\ell,k,t}(x_{\ell,k})$ (and 0 if the KPI indices do not match).

Let $\mathcal{C}$ be a set of contexts *(e.g. $\{normal, incident\}$). Each context $C \in \mathcal{C}$ fixes an* anchor map

$$b_C :\ \mathcal{I} \to \mathcal{T}, \qquad (\text{“preferred” term per KPI in context } C).$$

We abbreviate $c_C((\ell,k),t) := pCF(((\ell,k),t),((\ell,k),b_C(\ell,k))) \in [0,1]$.

Definition 13 (PFITIL$^{\mathrm{UD}}$ transform)**.** Fix contexts $C, C' \in \mathcal{C}$ and a threshold $\tau \in [0,1]$. Given a PFITIL quintuple $(P = \mathcal{I},\ v,\ Pv,\ pdf,\ pCF)$ at context C, define

$$U_{C \to C',\tau}(P, v, Pv, pdf, pCF) \ :=: \ (P, v, Pv, pdf^{UD}, pCF^{UD})$$

by

$$pdf^{UD}\big((\ell,k),((\ell,k),t)\big) = \begin{cases} 1 - \mu_{\ell,k,t}(x_{\ell,k}), & c_{C'}((\ell,k),t) \ \geq \tau, \\ \mu_{\ell,k,t}(x_{\ell,k}), & c_{C'}((\ell,k),t) \ < \tau, \end{cases}$$

and for all $a, b \in Pv$

$$pCF^{UD}(a,b) = \begin{cases} 0, & \{a,b\} = \{\,((\ell,k),t),\ ((\ell,k),b_{C'}(\ell,k))\,\} \text{ with } c_{C'}((\ell,k),t) \geq \tau, \\ pCF(a,b), & \text{otherwise.} \end{cases}$$

Whenever the contradiction of an attribute value t against the *new* anchor $b_{C'}(\ell,k)$ exceeds τ, its membership is flipped and its pairwise contradiction to the anchor is neutralized (reset to 0) after the switch.

Remark 2 (Plithogenicity and well-posedness)**.** For each $(\ell,k),t$ the mapping $pdf^{UD} \in [0,1]$ is well defined; pCF^{UD} remains symmetric with $pCF^{UD}(a,a) = 0$. Hence $(P, v, Pv, pdf^{UD}, pCF^{UD})$ is a valid Plithogenic Fuzzy Set ($s = t = 1$).

Example 14 (Peak-sale surge: cost anchor flips from *Low* to *High* (SO area))**.** *Scenario.* A 24/7 e-commerce site enters a peak-sale window. We switch context from $C =$ normal to $C' =$ incident with threshold $\tau = 0.8$. Three KPIs are used in Service Operation (SO): SLA compliance $s = 0.91$, autoscaling headroom $h = 0.18$ (fraction), and cloud spend rate $c = 5.0$ (k\$ /h).

Terms and memberships (triangular).

$$\begin{aligned}&\text{SLA: } H_s : (0.88, 0.95, 1.00), \quad M_s : (0.80, 0.88, 0.95);\\ &\text{Headroom: } M_h : (0.10, 0.20, 0.30);\\ &\text{Cost: } L_c : (0, 1.5, 3.0), \quad M_c : (2.5, 3.5, 4.5), \quad H_c : (4.0, 5.0, 6.0).\end{aligned}$$

Compute

$$\mu_{H_s}(0.91) = \frac{0.91 - 0.88}{0.95 - 0.88} = \frac{3}{7} \approx 0.4286, \quad \mu_{M_s}(0.91) = \frac{0.95 - 0.91}{0.95 - 0.88} = \frac{4}{7} \approx 0.5714;$$
$$\mu_{M_h}(0.18) = \frac{0.18 - 0.10}{0.20 - 0.10} = 0.80;$$
$$\mu_{L_c}(5.0) = 0, \quad \mu_{M_c}(5.0) = 0, \quad \mu_{H_c}(5.0) = 1.$$

Anchors and contradiction. Normal C: $b_C(\text{Cost}) = L_c$; Incident C': $b_{C'}(\text{Cost}) = H_c$ (speed over cost). Set

$$pCF(L_c, H_c) = 1, \quad pCF(M_c, H_c) = 0.5, \quad \text{others } = 0, \qquad \tau = 0.8.$$

Activation w.r.t. C': only L_c activates since $pCF(L_c, H_c) = 1 \geq \tau$. Thus the PFITIL$^{\text{UD}}$ flip gives

$$pdf^{UD}(L_c) = 1 - \mu_{L_c} = 1 - 0 = 1, \qquad pCF^{UD}(L_c, H_c) = 0.$$

All other memberships remain as above.

Rules and inference (Sugeno, $T = \min$).

$$\begin{aligned}&\mathbf{R1}: \quad (H_s) \wedge (M_h) \wedge (H_c) \implies z_1 = 4.6,\\ &\mathbf{R2}: \quad (M_s) \wedge (M_h) \wedge (H_c \ \vee \ M_c) \implies z_2 = 3.8.\end{aligned}$$

In C' after the PFITIL$^{\text{UD}}$ transform,

$$w_1^{UD} = \min\Big(\tfrac{3}{7},\, 0.8,\, 1\Big) = \tfrac{3}{7}, \qquad w_2^{UD} = \min\Big(\tfrac{4}{7},\, 0.8,\, \max(1, 0)\Big) = \tfrac{4}{7}.$$

Hence the SO maturity is

$$M_{\text{SO}}^{UD} = \frac{\frac{3}{7} \cdot 4.6 + \frac{4}{7} \cdot 3.8}{\frac{3}{7} + \frac{4}{7}} = \frac{13.8/7 + 15.2/7}{1} = \frac{29.0}{7} \approx 4.143.$$

This illustrates how, under a peak-sale *incident* context, the upside-down flip neutralizes the strong contradiction between L_c and the new anchor H_c and raises the incident-mode maturity.

Example 15 (Real-time freshness pivot: lag semantics invert (SO area))**.** *Scenario.* A data platform switches from batch mode to a real-time incident response. We move from C = batch to C' = real-time with $\tau = 0.8$. KPIs: ingestion throughput $q = 22$k rec/min, data staleness (lag) $\ell = 14$ min, cost $c = 2.8$ k\$ /h.

Terms and memberships (triangular).

$$\begin{aligned}&\text{Throughput: } H_q : (18, 24, 30), \ M_q : (12, 18, 24);\\ &\text{Lag: } L_\ell : (0, 5, 10), \ M_\ell : (8, 12, 16), \ H_\ell : (12, 20, 30);\\ &\text{Cost: } L_c : (0, 1.5, 3.0), \ M_c : (2.5, 3.5, 4.5), \ H_c : (4.0, 5.0, 6.0).\end{aligned}$$

Compute

$$\mu_{H_q}(22) = \frac{22 - 18}{24 - 18} = \frac{4}{6} = \frac{2}{3}, \quad \mu_{M_q}(22) = \frac{24 - 22}{24 - 18} = \frac{2}{6} = \frac{1}{3};$$
$$\mu_{L_\ell}(14) = 0, \quad \mu_{M_\ell}(14) = \frac{16 - 14}{16 - 12} = \frac{1}{2}, \quad \mu_{H_\ell}(14) = \frac{14 - 12}{20 - 12} = \frac{1}{4};$$
$$\mu_{L_c}(2.8) = \frac{3.0 - 2.8}{3.0 - 1.5} = \frac{2}{15} \approx 0.1333, \quad \mu_{M_c}(2.8) = \frac{2.8 - 2.5}{3.5 - 2.5} = \frac{3}{10} = 0.3, \quad \mu_{H_c}(2.8) = 0.$$

Anchors and contradiction. Batch C: $b_C(\text{Lag}) = H_\ell$; Real-time C': $b_{C'}(\text{Lag}) = L_\ell$ (freshness-first). Set

$$pCF(H_\ell, L_\ell) = 1, \quad pCF(M_\ell, L_\ell) = 0.5, \quad pCF(L_c, M_c) = 0.5, \quad \text{others } = 0, \qquad \tau = 0.8.$$

Activation (w.r.t. C'): only H_ℓ activates ($1 \geq \tau$). Then

$$pdf^{UD}(H_\ell) = 1 - \mu_{H_\ell} = 1 - \tfrac{1}{4} = \tfrac{3}{4}, \qquad pCF^{UD}(H_\ell, L_\ell) = 0,$$

and all other memberships are unchanged.

Rules and inference (Sugeno, $T = \min$).

$$\mathbf{R1}: \quad (H_q) \wedge (H_\ell) \wedge (M_c \ \vee \ L_c) \implies z_1 = 4.6,$$
$$\mathbf{R2}: \quad (M_q) \wedge (H_\ell) \implies z_2 = 3.2.$$

After the PFITIL$^{\mathrm{UD}}$ transform,

$$w_1^{UD} = \min\Big(\tfrac{2}{3},\ \tfrac{3}{4},\ \max(\tfrac{3}{10},\tfrac{2}{15})\Big) = \min\Big(\tfrac{2}{3},\ \tfrac{3}{4},\ \tfrac{3}{10}\Big) = \tfrac{3}{10},$$
$$w_2^{UD} = \min\Big(\tfrac{1}{3},\ \tfrac{3}{4}\Big) = \tfrac{1}{3}.$$

Hence

$$M_{\mathrm{SO}}^{UD} = \frac{\frac{3}{10}\cdot 4.6 + \frac{1}{3}\cdot 3.2}{\frac{3}{10}+\frac{1}{3}} = \frac{\frac{69}{50}+\frac{16}{15}}{\frac{19}{30}} = \frac{\frac{367}{150}}{\frac{19}{30}} = \frac{367}{95} \approx 3.863.$$

Here the upside-down flip raises the effective salience of *High Lag* (from 1/4 to 3/4) against the new real-time anchor, resulting in stronger activation of response rules and a higher real-time operations maturity.

Notation 5 (Inference layer (Sugeno form) and area/global maturity)**.** *Rules are of Sugeno type:*

$$\textit{Rule } r: \quad \bigwedge_{(\ell,k)\in A_r} \big((\ell,k) \textit{ is } t_{\ell,k}^{(r)}\big) \implies z_r \in [1,5].$$

With a t-norm $T \in \{\min, \cdot\}$, define the firing strength in context $C \to C'$ by

$$w_r^{UD} = T\Big(\ \{\, pdf^{UD}\big((\ell,k),((\ell,k),t_{\ell,k}^{(r)})\big)\,\}_{(\ell,k)\in A_r} \ \Big).$$

Area-level maturity and global maturity are the standard Sugeno weighted averages:

$$M_\ell^{UD} = \frac{\sum_{r\in R_\ell} w_r^{UD}\, z_r}{\sum_{r\in R_\ell} w_r^{UD}}, \qquad M_{\mathrm{global}}^{UD} = \sum_{\ell\in\mathcal{L}} \alpha_\ell\, M_\ell^{UD}, \quad \alpha_\ell \ge 0,\ \sum_\ell \alpha_\ell = 1.$$

Proposition 5 (Crisp reduction and FITIL reduction)**.** *(a) If $pCF \equiv 0$ or $\tau > 1$, then $U_{C\to C',\tau}$ acts as identity and M_ℓ^{UD} coincides with the ordinary fuzzy ITIL maturity M_ℓ.*
(b) If, additionally, every $\mu_{\ell,k,t} \in \{0,1\}$ and exactly one term fires per KPI, then M_ℓ^{UD} coincides with the crisp ITIL scoring.

Proof: (a) If $pCF \equiv 0$ then $c_{C'}((\ell,k),t) = 0 < \tau$ for any $\tau \le 1$, hence no flip occurs and $pdf^{UD} = pdf$, $pCF^{UD} = pCF$. The inference pipeline is unchanged. (b) With single-term activation, all rule weights are 0 or 1 as in the crisp mapping, hence the Sugeno average returns the crisp level. □

Proposition 6 (Idempotence on a fixed target context)**.** *For fixed C' and τ, $U_{C'\to C',\tau} \circ U_{C\to C',\tau} = U_{C\to C',\tau}$.*

Proof: After the first application, any activated pair $((\ell,k),t)$ vs. $((\ell,k),b_{C'}(\ell,k))$ has $pCF^{UD} = 0$, thus fails the activation test on the second application. Hence no further change occurs. □

Example 16 (Additional Real-life operations: switching from *cost-minimizing* mode to *incident* mode)**.** Consider the SO (Service Operation) area of a 24/7 online service with three KPIs measured last month:

$$\text{SLA compliance } s = 0.93, \qquad \text{MTTR } m = 2.8 \text{ h}, \qquad \text{Cloud spend rate } c = 4.2 \text{ k\$/h}.$$

Memberships (triangular):

$$\text{SLA: } H_s : (0.88, 0.96, 1.00),\ M_s : (0.80, 0.88, 0.96);$$
$$\text{MTTR: } L_m : (0, 2, 4),\ M_m : (2, 4, 6);$$
$$\text{Cost: } L_c : (0, 1.5, 3.0),\ M_c : (2.5, 3.5, 4.5),\ H_c : (4.0, 5.0, 6.0).$$

By the triangular formula,

$$\mu_{H_s}(0.93) = \frac{0.93-0.88}{0.96-0.88} = \frac{0.05}{0.08} = 0.625, \quad \mu_{M_s}(0.93) = \frac{0.96-0.93}{0.96-0.88} = \frac{0.03}{0.08} = 0.375;$$

$$\mu_{L_m}(2.8) = \frac{4-2.8}{4-2} = \frac{1.2}{2} = 0.6, \quad \mu_{M_m}(2.8) = \frac{2.8-2}{4-2} = \frac{0.8}{2} = 0.4;$$
$$\mu_{L_c}(4.2) = 0 \text{ (since } 4.2 \geq 3.0), \quad \mu_{M_c}(4.2) = \frac{4.5-4.2}{4.5-3.5} = \frac{0.3}{1.0} = 0.3, \quad \mu_{H_c}(4.2) = \frac{4.2-4.0}{5.0-4.0} = \frac{0.2}{1.0} = 0.2.$$

Contradiction and anchors. Define DCF on cost terms with $pCF(L_c, H_c) = 1$, $pCF(M_c, H_c) = 0.5$, $pCF(L_c, M_c) = 0.5$ and symmetry; other KPI terms have zero contradiction to their anchors. Set threshold $\tau = 0.8$.

Normal context C = normal anchors: $b_C(\text{SLA}) = H_s$, $b_C(\text{MTTR}) = L_m$, $b_C(\text{Cost}) = L_c$ (cost-minimizing). *Incident context* C' = incident anchors: $b_{C'}(\text{SLA}) = H_s$, $b_{C'}(\text{MTTR}) = L_m$, $b_{C'}(\text{Cost}) = H_c$ (speed-over-cost).

Upside-Down switch $U_{C\to C',\tau}$. Activation set on *cost* w.r.t. $b_{C'} = H_c$ is

$$A_\tau(b_{C'}) = \{L_c\} \quad \text{since} \quad pCF(L_c, H_c) = 1 \geq 0.8, \quad pCF(M_c, H_c) = 0.5 < 0.8.$$

Hence only the membership of L_c flips; others remain:

$$pdf^{UD}(L_c) = 1 - \mu_{L_c} = 1 - 0 = 1, \qquad pdf^{UD}(M_c) = \mu_{M_c} = 0.3, \qquad pdf^{UD}(H_c) = \mu_{H_c} = 0.2.$$

Contradiction is reset on the flipped pair: $pCF^{UD}(L_c, H_c) = 0$.

Rules (Sugeno, $T = \min$). Use a small, context-agnostic rule base that refers to an *alignment literal*:

$$\text{Align_Cost} := \text{“Cost term equals current anchor”}.$$

Operationally we implement it as the anchor-term membership (here, H_c in incident, L_c in normal). Consider

$$\textbf{R1}: \ (H_s) \wedge (L_m) \wedge (\text{Align_Cost}) \implies z_1 = 4.5,$$
$$\textbf{R2}: \ (H_s) \wedge (L_m) \wedge (\neg\text{Align_Cost}) \implies z_2 = 3.6.$$

Normal C: Align_Cost $= \mu_{L_c} = 0$, so $w_1^{(C)} = \min(0.625, 0.6, 0) = 0$, $w_2^{(C)} = \min(0.625, 0.6, 1) = 0.6$, and thus

$$M_{\text{SO}}^{(C)} = \frac{0.6 \cdot 3.6}{0.6} = 3.6.$$

Incident C' *after* $U_{C\to C',\tau}$: Align_Cost $= \mu_{H_c} = 0.2$ (anchor is H_c), hence

$$w_1^{(C')} = \min(0.625, 0.6, 0.2) = 0.2, \qquad w_2^{(C')} = \min(0.625, 0.6, 0.8) = 0.6,$$

and

$$M_{\text{SO}}^{(C')} = \frac{0.2 \cdot 4.5 + 0.6 \cdot 3.6}{0.2 + 0.6} = \frac{0.900 + 2.160}{0.8} = \frac{3.060}{0.8} = 3.825.$$

Thus switching to the *incident* context increases the SO maturity from 3.6 to 3.825 because the alignment target for cost reverses. Note also the explicit upside-down flip on the contradictory term: $\mu_{L_c} = 0 \ \mapsto \ pdf^{UD}(L_c) = 1$ since $pCF(L_c, H_c) = 1 \geq \tau$.

2.3| Two-Mode De-Plithogenication in Plithogenic Fuzzy ITIL

Two-mode De-Plithogenication in PFITIL resolves KPI-term contradictions by activation against anchors: Mode 0 neutralizes without flipping; Mode 1 inverts then neutralizes, producing context-consistent maturity assessments.

Notation 6. *Assume a fixed PFITIL tuple* (P, v, Pv, pdf, pCF)*, an* anchor map $b : P \to \mathcal{T}$ *(preferred term per KPI instance), a threshold* $\tau \in [0, 1]$*, and a* mode selector

$$\mathcal{M}: \ \big\{\{a, b(u)\} \subseteq Pv \mid u \in P, \ a = ((u), t)\big\} \longrightarrow \{0, 1\},$$

interpreted as

$$\mathcal{M}(\{a, b(u)\}) = \begin{cases} 0 & \text{Neutralize-only } \textit{(apparent contradiction);} \\ 1 & \text{Invert+Neutralize } \textit{(genuine contradiction).} \end{cases}$$

Write $c(a, b) := pCF(a, b)$ *and define the* activation indicator

$$\mathsf{Act}(a \mid b, \tau) \ := \ \mathbf{1}\big[c(a, b) \geq \tau\big].$$

Definition 2.23 (Two-mode De-Plithogenication operator)**.** For each KPI instance $u \in P$ and each attribute value $a = ((u), t) \in Pv$, define

$$\mathrm{D}^{\mathrm{tm}}_{b,\tau,\mathcal{M}}(P, v, Pv, pdf, pCF) \;:=:\; \big(P, v, Pv, \widehat{pdf}, \widehat{pCF}\big),$$

with

$$\widehat{pdf}(u,a) \;=\; \begin{cases} 1 - pdf(u,a), & \mathsf{Act}(a \mid b(u), \tau) = 1 \text{ and } \mathcal{M}(\{a, b(u)\}) = 1, \\ pdf(u,a), & \text{otherwise,} \end{cases}$$

and

$$\widehat{pCF}(x,y) \;=\; \begin{cases} 0, & \exists\, u \in P,\ a = ((u), t) :\ \{x,y\} = \{a, b(u)\} \ \wedge\ \mathsf{Act}(a \mid b(u), \tau) = 1, \\ pCF(x,y), & \text{otherwise,} \end{cases}$$

for all $x, y \in Pv$.

Remark 3. 1) $\widehat{pdf}$ remains in $[0,1]$ and $\widehat{pCF}$ is symmetric with zeros on the diagonal, so the result is again a valid PFITIL structure.

2) Idempotence on stabilized pairs: if $\mathsf{Act}(a \mid b(u), \tau) = 1$ in the first application, then the corresponding contradiction is reset to 0, hence a second application makes no further change on that pair.

Example 17 (Incident triage with mixed contradictions (SO area, one genuine and one apparent))**.** *Setup.* Consider three KPIs in Service Operation for a web platform (monthly aggregates): error rate $e = 4.2\%$, mean time to repair $m = 3.8\,\mathrm{h}$, cloud cost $c = 2.6\,\mathrm{k\$/h}$. For each KPI use triangular terms:

Error (%): $L_e : (0,1,2)$, $M_e : (1.5,3,5)$, $H_e : (4,7,10)$.

MTTR (h): $L_m : (0,2,4)$, $M_m : (3,5,7)$.

Cost (k$/h): $L_c : (0,1.5,3)$, $M_c : (2.5,3.5,4.5)$, $H_c : (4,5,6)$.

Raw memberships (by triangular formula).

$$\mu_{L_e}(4.2) = 0, \quad \mu_{M_e}(4.2) = \frac{5-4.2}{5-3} = \frac{0.8}{2} = 0.4, \quad \mu_{H_e}(4.2) = \frac{4.2-4}{7-4} = \frac{0.2}{3} \approx 0.0667;$$

$$\mu_{L_m}(3.8) = \frac{4-3.8}{4-2} = \frac{0.2}{2} = 0.1, \qquad \mu_{M_m}(3.8) = \frac{3.8-3}{5-3} = \frac{0.8}{2} = 0.4;$$

$$\mu_{L_c}(2.6) = \frac{3.0-2.6}{3.0-1.5} = \frac{0.4}{1.5} \approx 0.2667, \quad \mu_{M_c}(2.6) = \frac{2.6-2.5}{3.5-2.5} = 0.1, \quad \mu_{H_c}(2.6) = 0.$$

Anchors & contradictions. Desired anchors (policy): $b(e) = L_e$ (low errors), $b(m) = L_m$ (low MTTR), $b(c) = L_c$ (low cost). Let the threshold be $\tau = 0.8$. Domain knowledge (contradiction magnitudes):

$$c(M_m, L_m) = 0.9 \quad \text{(genuine tension)},$$
$$c(H_e, L_e) = 0.85 \quad \text{(apparent spike due to a noisy probe)},$$
$$c(M_c, L_c) = 0.5 \text{ (sub-threshold)}.$$

Choose the two-mode policy

$$\mathcal{M}(\{M_m, L_m\}) = 1 \quad \text{(Invert+Neutralize)},$$
$$\mathcal{M}(\{H_e, L_e\}) = 0 \quad \text{(Neutralize-only)}.$$

Apply $\mathrm{D}^{\mathrm{tm}}_{b,\tau,\mathcal{M}}$. Activated pairs are $\{M_m, L_m\}$ and $\{H_e, L_e\}$ (both $\geq \tau$). Thus

$$\widehat{pdf}(m, M_m) = 1 - \mu_{M_m} = 1 - 0.4 = 0.6, \qquad \widehat{pCF}(M_m, L_m) = 0,$$
$$\widehat{pdf}(e, H_e) = \mu_{H_e} = 0.0667 \quad \text{(no flip)}, \qquad \widehat{pCF}(H_e, L_e) = 0.$$

All other memberships remain as computed above.

Rule base (Sugeno, $T = \min$).

R1 : $(L_e) \wedge (L_m) \wedge (L_c) \;\Rightarrow\; z_1 = 4.4$, **R2** : $(M_e) \vee (M_m) \;\Rightarrow\; z_2 = 3.2$, **R3** : $(H_e) \;\Rightarrow\; z_3 = 2.2$.

($\vee$ is implemented by max.)

Before de-plithogenication.

$$w_1 = \min(0,\, 0.1,\, 0.2667) = 0, \quad w_2 = \max(0.4,\, 0.4) = 0.4, \quad w_3 = 0.0667.$$
$$M_{\mathrm{SO}} = \frac{0.4 \cdot 3.2 + 0.0667 \cdot 2.2}{0.4 + 0.0667} = \frac{1.28 + 0.1467}{0.4667} \approx 3.06.$$

After two-mode de-plithogenication. Only M_m flips to 0.6; hence

$$w_1^\star = 0, \quad w_2^\star = \max(0.4,\, 0.6) = 0.6, \quad w_3^\star = 0.0667,$$
$$M_{\mathrm{SO}}^\star = \frac{0.6 \cdot 3.2 + 0.0667 \cdot 2.2}{0.6 + 0.0667} = \frac{1.92 + 0.1467}{0.6667} \approx 3.10.$$

Interpretation. The operator: 1) Mode 1 treats the MTTR conflict as genuine and flips its membership (penalizing the variance from the low-MTTR anchor), and 2) Mode 0 treats the transient high-error reading as an apparent contradiction, leaving its membership intact but removing its antagonism with the target low-error anchor. The SO maturity rises from ≈ 3.06 to ≈ 3.10, reflecting a more faithful post-triage assessment.

Example 18 (Blue/Green rollback (ST area): one genuine and one apparent contradiction)**.** *Context.* Service Transition (ST) during a Blue/Green deployment. KPIs: change success rate CSR $= 0.88$, deployment lead time $L = 5.5\,\mathrm{h}$, error-budget consumption $E = 0.12$.

Terms (triangular).

CSR : $H_c : (0.85, 0.95, 1.00),\ M_c : (0.75, 0.85, 0.95)$;
Lead time: $L_\ell : (0, 2, 6),\ M_\ell : (4, 8, 12)$;
Err. budget: $L_e : (0, 0.05, 0.10),\ M_e : (0.08, 0.15, 0.25),\ H_e : (0.20, 0.35, 0.50)$.

Raw memberships (triangular formula).

$$\mu_{H_c}(0.88) = \frac{0.88 - 0.85}{0.95 - 0.85} = 0.3,$$
$$\mu_{M_c}(0.88) = \frac{0.95 - 0.88}{0.95 - 0.85} = 0.7;$$
$$\mu_{L_\ell}(5.5) = \frac{6 - 5.5}{6 - 2} = \frac{1}{8} = 0.125,$$
$$\mu_{M_\ell}(5.5) = \frac{5.5 - 4}{8 - 4} = \frac{3}{8} = 0.375;$$
$$\mu_{L_e}(0.12) = 0,$$
$$\mu_{M_e}(0.12) = \frac{0.12 - 0.08}{0.15 - 0.08} = \frac{4}{7} \approx 0.5714,$$
$$\mu_{H_e}(0.12) = 0.$$

Anchors and contradictions (threshold $\tau = 0.8$). Anchors: $b(\mathrm{CSR}) = H_c$, $b(L) = L_\ell$, $b(E) = L_e$. Contradictions: $c(M_\ell, L_\ell) = 0.9$ (genuine, Mode 1), $c(M_c, H_c) = 0.85$ (apparent, Mode 0), $c(M_e, L_e) = 0.6$ (sub-threshold).

Apply the two-mode operator $\mathrm{D}^{\mathrm{tm}}_{b,\tau,\mathcal{M}}$. Activated pairs: $\{M_\ell, L_\ell\}$ and $\{M_c, H_c\}$. Hence

$$\widehat{\mu}(M_\ell) = 1 - \mu_{M_\ell} = 1 - \tfrac{3}{8} = \tfrac{5}{8} = 0.625 \quad \text{(flip, Mode 1)}, \qquad \widehat{\mu}(M_c) = \mu_{M_c} = 0.7 \quad \text{(no flip, Mode 0)},$$

and $pCF(M_\ell, L_\ell) = pCF(M_c, H_c) = 0$ after neutralization. Other memberships unchanged.

Rule base (Sugeno, $T = \min$, $\vee = \max$).

R1 : $(H_c) \wedge (L_\ell) \wedge (L_e \vee M_e) \ \Rightarrow\ z_1 = 4.4$, **R2** : $(M_c) \wedge (M_\ell) \ \Rightarrow\ z_2 = 3.2$, **R3** : $(M_\ell) \wedge (M_e) \ \Rightarrow\ z_3 = 3.0$.

Before operator.

$$w_1 = \min(0.3,\, 0.125,\, \max(0,\, \tfrac{4}{7})) = 0.125, \quad w_2 = \min(0.7,\, 0.375) = 0.375, \quad w_3 = \min(0.375,\, \tfrac{4}{7}) = 0.375.$$
$$M_{\mathrm{ST}} = \frac{0.125 \cdot 4.4 + 0.375 \cdot 3.2 + 0.375 \cdot 3.0}{0.125 + 0.375 + 0.375} = \frac{\frac{23}{8}}{\frac{7}{8}} = \frac{23}{7} \approx 3.2857.$$

After operator.

$$w_1^\star = 0.125, \qquad w_2^\star = \min(0.7,\, 0.625) = 0.625, \qquad w_3^\star = \min(0.625,\, \tfrac{4}{7}) = \tfrac{4}{7}.$$

$$M_{\mathrm{ST}}^\star = \frac{0.125 \cdot 4.4 + 0.625 \cdot 3.2 + (\frac{4}{7}) \cdot 3.0}{0.125 + 0.625 + \frac{4}{7}} = \frac{\frac{597}{140}}{\frac{37}{28}} = \frac{597}{185} \approx 3.227.$$

Interpretation. Mode 1 treats *lead-time Medium* as a genuine contradiction to the low-lead-time anchor and flips it, increasing the activation of moderate-scoring rules and lowering maturity slightly; Mode 0 neutralizes the transient CSR medium vs. high tension without flipping, preventing an overreaction to an apparent conflict.

Example 19 (Autoscaling vs. latency pivot (SO/SD): genuine latency, apparent CPU underutilization)**.** *Context.* Real-time service under bursty load. KPIs: p95 latency $L_{95} = 320\,\mathrm{ms}$, average CPU utilization $U = 0.25$, cloud cost $C = 2.1\,\mathrm{k\$/h}$.

Terms (triangular).

$$\begin{aligned}&\text{Latency: } L_L : (0, 100, 200),\ M_L : (150, 250, 350),\ H_L : (300, 450, 600);\\ &\text{CPU: } L_U : (0, 0.3, 0.5),\ M_U : (0.4, 0.6, 0.75),\ H_U : (0.7, 0.85, 1.0);\\ &\text{Cost: } L_C : (0, 1.5, 3.0),\ M_C : (2.5, 3.5, 4.5),\ H_C : (4.0, 5.0, 6.0).\end{aligned}$$

Raw memberships.

$$\mu_{L_L}(320) = 0, \quad \mu_{M_L}(320) = \frac{350 - 320}{350 - 250} = \frac{3}{10} = 0.3, \quad \mu_{H_L}(320) = \frac{320 - 300}{450 - 300} = \frac{2}{15} \approx 0.1333;$$

$$\mu_{L_U}(0.25) = \frac{0.25 - 0}{0.3 - 0} = \frac{5}{6} \approx 0.8333, \quad \mu_{M_U}(0.25) = 0, \quad \mu_{H_U}(0.25) = 0;$$

$$\mu_{L_C}(2.1) = \frac{3.0 - 2.1}{3.0 - 1.5} = \frac{3}{5} = 0.6, \quad \mu_{M_C}(2.1) = 0, \quad \mu_{H_C}(2.1) = 0.$$

Anchors and contradictions (threshold $\tau = 0.8$). Anchors: $b(L) = L_L$ (low latency), $b(U) = M_U$ (balanced CPU), $b(C) = L_C$ (low cost). Contradictions: $c(H_L, L_L) = 0.95$ (genuine, Mode 1), $c(L_U, M_U) = 0.85$ (apparent, Mode 0). Others $= 0$.

Apply the two-mode operator $\mathrm{D}^{\mathrm{tm}}_{b,\tau,\mathcal{M}}$. Activated pairs: $\{H_L, L_L\}$ (flip) and $\{L_U, M_U\}$ (no flip). Thus

$$\widehat{\mu}(H_L) = 1 - \mu_{H_L} = 1 - \tfrac{2}{15} = \tfrac{13}{15} \approx 0.8667, \qquad \widehat{\mu}(L_U) = \mu_{L_U} = \tfrac{5}{6},$$

and $pCF(H_L, L_L) = pCF(L_U, M_U) = 0$ after neutralization.

Rule base (Sugeno, $T = \min$, $\vee = \max$).

$$\mathbf{R1}:\ (L_L) \wedge (M_U \vee L_U) \wedge (L_C)\ \Rightarrow\ z_1 = 4.4, \quad \mathbf{R2}:\ (H_L)\ \Rightarrow\ z_2 = 2.6, \quad \mathbf{R3}:\ (L_U) \wedge (L_C)\ \Rightarrow\ z_3 = 3.4.$$

Before operator.

$$w_1 = \min(0,\, \max(0, \tfrac{5}{6}),\, \tfrac{3}{5}) = 0, \quad w_2 = \tfrac{2}{15}, \quad w_3 = \min(\tfrac{5}{6},\, \tfrac{3}{5}) = \tfrac{3}{5}.$$

$$M_{\mathrm{SO}} = \frac{(\frac{2}{15}) \cdot 2.6 + (\frac{3}{5}) \cdot 3.4}{\frac{2}{15} + \frac{3}{5}} = \frac{\frac{179}{75}}{\frac{11}{15}} = \frac{179}{55} \approx 3.2545.$$

After operator.

$$w_1^\star = 0, \qquad w_2^\star = \tfrac{13}{15}, \qquad w_3^\star = \tfrac{3}{5},$$

$$M_{\mathrm{SO}}^\star = \frac{(\frac{13}{15}) \cdot 2.6 + (\frac{3}{5}) \cdot 3.4}{\frac{13}{15} + \frac{3}{5}} = \frac{\frac{322}{75}}{\frac{22}{15}} = \frac{161}{55} \approx 2.9273.$$

Interpretation. Mode 1 flips the *High Latency* membership against the low-latency anchor, greatly increasing the weight of the bad-latency rule and lowering maturity; Mode 0 neutralizes the CPU underutilization vs. target tension without flipping, avoiding punitive scoring for a benign (policy-intended) headroom.

3| Conclusion

In this paper, we investigated *Plithogenic Fuzzy ITIL*, which extends Fuzzy ITIL by incorporating the structure of a Plithogenic Fuzzy Set. We expect that future work will include quantitative analyses, practical applications, and case studies exploring the effectiveness of this framework. Furthermore, it is also expected that research will advance on extended frameworks employing HyperFuzzy Sets [30], [31], Rough Sets [32], [33], Soft Sets [34-36], and Neutrosophic Sets [15], [16], [37].

Acknowledgments

The authors wish to thank all colleagues and mentors whose feedback and encouragement enriched this work. We are grateful to the community of researchers whose foundational contributions informed our developments. Special appreciation goes to the institutions that offered resources and technical infrastructure throughout this project.

Funding

No external funding or financial support was provided for this study.

Data Availability

This manuscript presents purely conceptual work without empirical data. Scholars interested in these ideas are invited to undertake experimental or case-study research to substantiate and extend the proposed frameworks.

Conflicts of Interest

The authors declare that there are no competing interests concerning the content or publication of this article.

Ethics Approval and Consent to Participate

This paper involves no human or animal subjects and thus did not require ethics committee review or approval.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Disclaimer

The theoretical models and propositions herein have not yet been subjected to practical validation. Readers should independently verify all citations and be aware that inadvertent inaccuracies may remain. The opinions expressed are those of the authors and do not necessarily represent the views of affiliated organizations.

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